Iterative modeling, migration and inversion (IMMI): evaluating the well-calibration technique to scale the gradient in the FWI process

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Summary

Iterative modeling, migration and inversion (IMMI) aims to incorporate standard processing techniques into the process of full waveform inversion (FWI). Within IMMI, depth migration method may be used to obtain the gradient, in contrast to standard FWI which uses a two-way reverse time migration (RTM). Another aspect of the IMMI approach is the use of well-calibration to scale the gradient, rather than applying a line search to find the scalar or an approximation of the inverse Hessian matrix. We examine with synthetic examples the performance of IMMI in circumstances of progressively increasing geological complexity. We find consistently low errors nearby the well-calibration location, even in the most complex settings. This suggests that the gradient obtained by applying a migration method other than RTM, though less wave-theoretically complete, points in the correct direction in order to minimize an FWI-like objective function, and that well-calibration provides a working approach for scaling. These refinements of FWI may be important enablers for application of waveform inversion in reservoir characterization, where we may have many control-wells, and we may wish to extend our approach to the determination of several elastic and/or rock properties. We find that well-calibration scales the updates properly up to what we refer to as moderate lateral velocity changes.

Introduction

Lailly (1983) and Tarantola (1984) provided the mathematical foundations for full waveform seismic inversion. They showed that FWI and migration are strongly linked, in what Margrave et al. (2010) called the fundamental theorem of FWI, which is summarized in Equation 1.

$$\delta v(x,z) = \lambda \nabla_v \phi_k(x,z,w) = \lambda \int \sum_{s,r} \omega^2 \hat{\Psi}_s(x,z,\omega)\,\delta\hat{\Psi}^*_{r(s),k}(x,z,\omega)\,d\omega \qquad (1)$$

where δv is the velocity update, λ is a scalar constant, ∇_v is the gradient with respect to the velocity model v, ϕ_k is the objective function for iteration k, ω is angular frequency, $\hat{\Psi}_s$ is a model of the source wavefield for source *s* propagated to all (x, z), $\delta\hat{\Psi}^*_{r(s),k}$ is the kth data residual for source *s* back propagated to all (x, z), and * means complex conjugation. The residual δΨ is the difference between the observed data and the modeled data. The objective function measures the difference between the recorded data and the modeled data at the kth iteration (equation 2).

$$\phi_k = \sum_{s,r} (\Psi - \Psi_k)^2 \qquad (2)$$

The gradient in Equation 1 can be written in the time domain as:

$$\nabla_v \phi_k(x,z) = -\sum_{s,r} \int \partial_t \psi_s(x,z,t)\,\partial_t \delta\psi_{r(s),k}(x,z,T-t)\,dt \qquad (3)$$

where T denotes record length. Equation 3 says that the gradient of the objective function is formed by correlating the time-reversed residuals propagated into the medium with the source field propagated into the medium. This is the core of FWI. The gradient is the element that contains the direction of the velocity update in the minimization scheme. The other element is the inverse Hessian or an approximation of it. If the inverse Hessian is replaced by a scalar λ, the mathematical effort is reduced to the gradient or steepest-descent method. λ scales the gradient to be converted into a velocity perturbation. λ is commonly estimated by a line-search method, which requires an extra forward modeling per shot (Virieux and Operto, 2009), doing the process more expensive.

FWI is an iterative cycle that involves four main steps, shown in Figure 1 (Margrave et al., 2010).

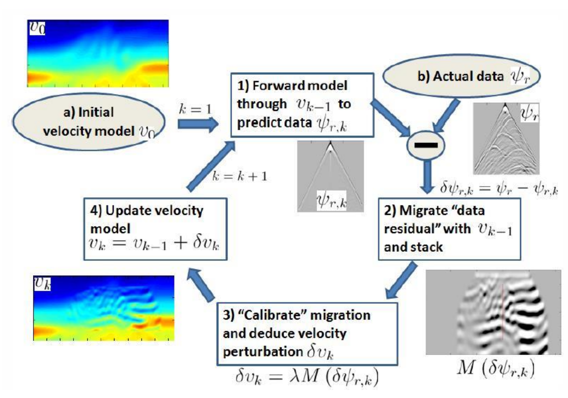

FIG. 1. The cycle of FWI (Margrave et al., 2010)

The first step consists in generating synthetic seismic data (predicted data $\Psi_{r,k}$) from an initial model v_o and the calculation of the data residual $\delta\Psi_{r,k} = \Psi_r - \Psi_{r,k}$. The second steps involves the pre-stack depth migration using the current velocity model of the data residual and stack to obtain $M(\delta\Psi_{r,k})$. This step provides the gradient or update direction. The third step is scaling or calibrating the gradient by aplying λ, which produces the velocity perturbation δv_k. The last step is updating the velocity model $v_k = v_{k-1} + \delta v_k$ that will be used in the next iteration.

Iterative Modeling, Migration and Inversion (IMMI), introduced by Margrave et al. (2012), was proposed as an alternative to "classical" FWI which involved tools already available and widely applied by the industry. The key IMMI innovations are the use of any depth migration method in place of RTM, and the incorporation of well information to scale the gradient. The authors further suggested that using a deconvolution imaging condition, instead of the correlation type generally employed, may produce updates similar those obtained by preconditioning with the main diagonal elements of the inverse Hessian, which is a gain correction, as illustrated by Shin et al. (2001). Pan et al. (2014) applied the IMMI method, compared the crosscorrelation and deconvolution imaging conditions, and showed that using a deconvolution-based gradient can compensate the geometrical spreading.

Following the IMMI approach, we used a phase-shift plus interpolation (PSPI) migration method (one-way wave migration) with a deconvolution imaging condition to obtain the gradient. PSPI, introduced by Gazdag and Sguazzero (1984), allows selecting a range of frequencies of interest, which is very convenient to explore frequency-based (i.e., multiscale) strategies in FWI, wherein the inversion is started using low frequencies and then higher frequencies are progressively included, to avoid local minima (Pratt, 1999). We will follow this strategy. The scale λ in Equation 1 takes the form of a match filter that equates the size of the gradient to the size of the velocity residual in a well location. The velocity residual is the difference between the well velocity and the current velocity model.

Method

The observed shots for this experiment are idealized version of the ones that would be recorded on the field. We generated these shots with an acoustic finite-difference algorithm to propagate the wavefield. A minimum phase wavelet with a dominant frequency of 20 Hz was used as seismic source. The sources are placed every 50 m from 2100 to 9250 m, giving 144 shots in total. Receiver stations are located along the whole model every 10 m, and all of them were kept alive for each shot.

First iteration

The initial velocity model was generated by applying a Gaussian smoother 290 meters wide to the true velocity model. The initial velocity model provides no more than 2 Hz of geological information, while the true velocity model mainly contains information between 1 and 30 Hz, with the main events around 12 Hz. The seismic data have a dominant frequency of roughly 15 Hz and provide information between 7 and 25 Hz. There is a gap between 2 and 5 Hz, where neither the initial model nor the seismic data contribute. Modeled shots were generated by using the initial model. The difference between the observed and the modeled shots is the data residual. We obtain a data residual per shot, which are migrated in depth with the PSPI method, which permits us to limit the process to a specific frequency range. We used frequencies between 1 and 5 Hz for the first iteration. A mute, before stacking the residuals, is commonly applied to avoid migration artifacts. The result of stacking the migrated data residuals is the gradient.

The next step is to scale or calibrate the gradient. We use well C to perform this process (Figure 3). The well calibration technique was described by Margrave et al. (2010). Figure 2 shows the calibration process. Firstly, the difference, ***δvel***, between the well and model velocities is calculated. The second step is to estimate the amplitude scalar ***a*** and the phase rotation ***ϕ*** that optimally match the gradient trace ***g*** to ***δvel***. The scalar ***a*** is found such that the difference between ***δve***l and ***ag*** is minimized by least squares. Finally, a convolutional match filter is obtained incoporporating ***a*** and ***ϕ***. This match filter is applied to every gradient trace in order to obtain the velocity update.

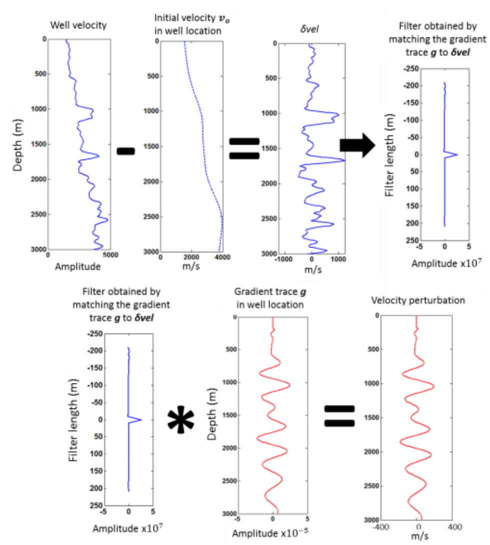

FIG. 2. Well calibration for the first iteration.

More iterations

The inputs which must be supplied for the subsequent iterations are the frequency range to be used, and the updated velocity model. The frequency range was increased by 1 Hz in each iteration. We stopped the inversion at the 10^{th} iteration because in our experiments the error in the model does not decrease anymore after that point.

Examples

We evaluated the performance of the well calibration technique in three different geological settings shown in Figure 3. Model 1 is the simplest model we consider, consisting of horizontal layers. The inversion is able to recover the most important features of the subsurface, including the low velocity body at a 2500 m depth, which is not present in the calibration well. The error is consistently low across the model. When moderate lateral velocities are found, such as in Model 2, the error across the model still decreases in each iteration, giving the best result around the calibration well. For this case, the resulting inverted model captures the main features and amplitudes of the true model, and again it is possible to identify the low velocity body enclosed in the anticline at 2500 m depth. In the presence of strong lateral velocity changes, such as in Model 3, the inversion produces good results in the vicinity of the calibration well, but the error increases quite strongly as we move away from the well, especially in the zones of the high velocity bodies.

Figure 4 illustrates the results when more than one well are incorporated to scale the gradient in constructing Model 3. Wells A, B and C were used to obtain an average calibration filter that was applied to scale the gradient. As we include more wells, the error across the model decreases. If more than one well is available, more options arise: for example, a spatial-varying filter can be estimated.

The match filter used for the experiments above was designed over the whole depth interval from zero to 3000 m. A more realistic experiment is shown in Figure 5,

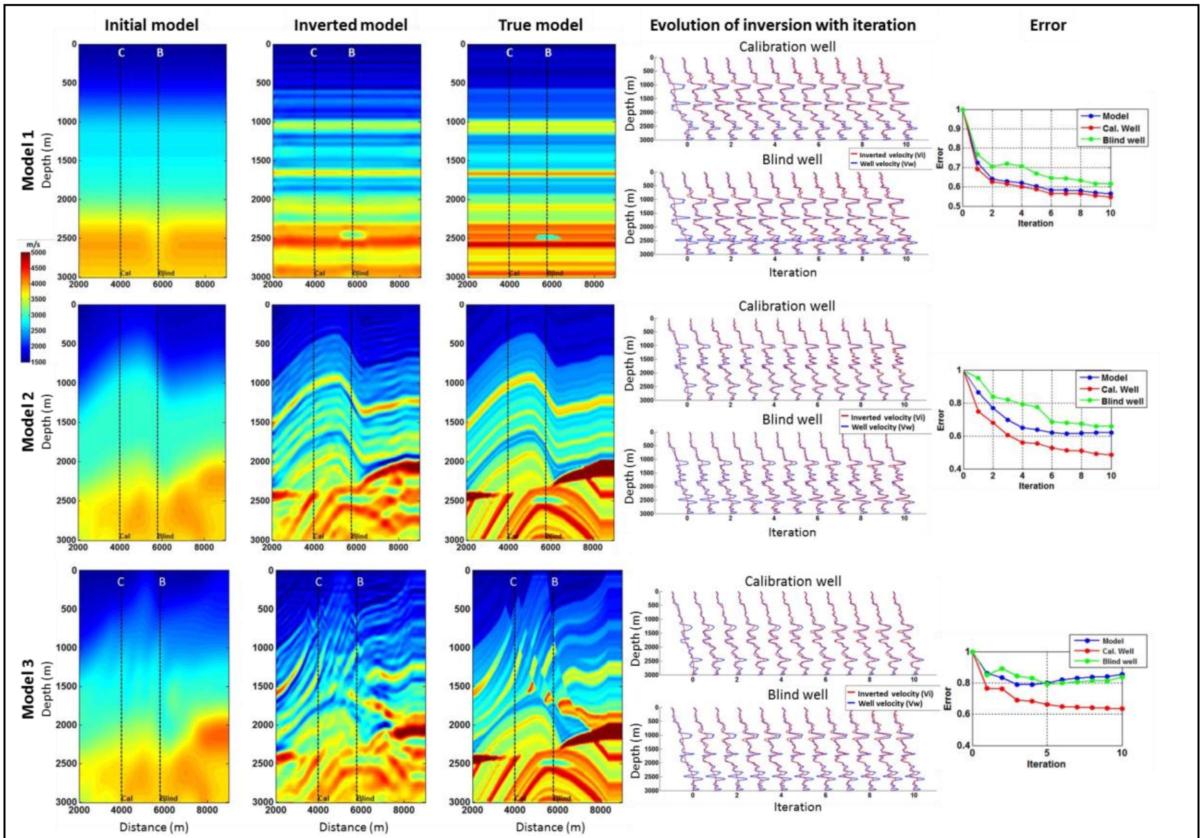

FIG. 3. Comparison among initial, inverted and true velocity models. The calibration and blind wells are C and B, respectively. The evolution of the inverted trace, from the initial model to iteration 10, shows an excellent performance in the calibration well for the three models, and the normalized error is consistenly low at this location. The error tends to increase with stronger lateral velocity variations and as we move away the calibration well.

wherein different depth intervals were selected to obtain the calibration filter. We note that the inverted trace is better in the depth zone where the filter is estimated. Following this observation, we tested a depth-varying calibration filter. The result is illustrated in the rightmost panel of Figure 5. The depth-varying filter provides superior results in comparison to those derived using a stationary filter. The example in the middle panel of Figure 5, where we use a depth interval from 1000-2250 m, exhibits poor inversion performance in the shallower part. This result is a reminder of the importance of sufficient well information when applying this technique.

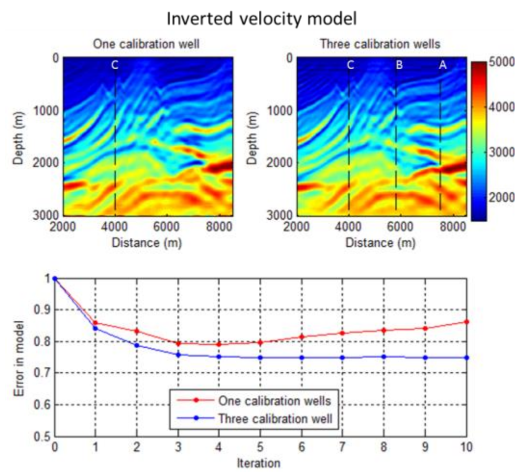

FIG. 4. Calibration with more than one well.

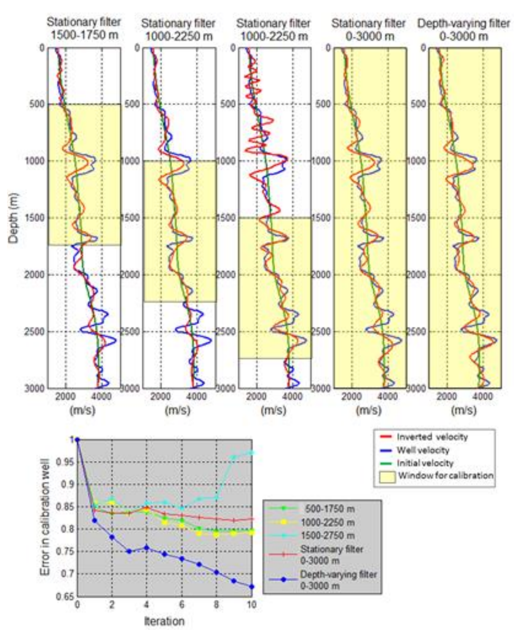

FIG. 5 Depth-varying calibration filter.

Conclusions

The gradient, calculated with a one-way wave migration method (PSPI) under a deconvolution imaging condition, points in the correct direction in order to minimize the objective function in as per the IMMI scheme. We showed that the use of well information to calibrate the gradient produces a velocity perturbation to update the model which reduces model error effectively in several benchmark examples. This was confirmed by the consistently low error at the well location, even for the most complex of the geological models. Well calibration satisfactorily performs in the presence of moderate lateral velocity changes, such as in Model 1 and 2. The error decreases in each iteration as we go to higher frequencies, and the main geological features of the subsurface are captured. When we have strong lateral velocity variations, such as in the Marmousi model, the inversion works properly in the shallow part, and is able to recover the main futures in the deeper part. However, the velocity tends to be underestimated as we go to deeper zones. A depth-varying calibration filter helps to overcome this issue. We found that well calibration can be applied in complex settings, providing that the well is representative of the geology of the area of interest. The results suggest that a calibration filter that varies horizontally (providing more control wells) and with depth, is a worthy option to obtain better velocity updates in the FWI process.

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